NGGM DATA EXCHANGE REFERENCE DOCUMENT

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1. INTRODUCTION

1.1 Scope

This document is submitted in partial fulfillment of WP 2310 of the Next Generation Gravity Mission (NGGM) study [AD-1][AD-2][AD-3].

The document shall serve as a reference for the NGGM science team for interpreting the data generated by the end-to-end (E2E) simulator.

1.2 Document description

- Chapter 3 describes the names of the output files;
- Chapter 4 defines the reference frames;
- Chapter 5 deals with quaternion definition and rotation matrix.

2. DOCUMENTS

2.1 Applicable Documents

[AD-1] Assessment of a Next Generation Gravity Mission to monitor the variations of Earth's gravity field, Statement of Work, EOP-SF/2008-09-1334, Issue 2, 20 November 2008, Appendix 1 to AO/1-5914/09/NL/CT

[AD-2] Special Conditions of Tender, Appendix 3 to AO/1-5914/09/NL/CT

[AD-3] ESTEC Contract Nº 22643/09/NL/AF.

3. OUTPUT FILES

Every simulation is identified with a name containing the type of orbit, the number of simulated days and the date (6 digits format) in which the simulation was run. For example, for a simulation of 5 days run on 21 December 2009 with a polar orbit, the name will be "polar_5days_211209".

The files generated by the E2E simulator have extension "*.dat". For each data file there is a header file listing the variables contained in the data file. The name of the header file starts with an "H" followed by the name of the data file it refers to.

More specific details about the simulation are written in a "readme" file with the same name of the simulation (for example: "readme_ polar_5days_211209.txt"). The details include:

- the simulation duration
- the type of orbit
- the static gravity field order
- the initial conditions
- which controls were activated (attitude, formation, drag free)
- if the variables were filtered and what type of filter was used
- the list of the output files
- the list of the header files
- if and how the files were compressed
- the date in which the simulation was run.

4. REFERENCE FRAMES DEFINITION

For convention, the satellite emitting the laser beam is denoted as S1 (satellite 1), while the satellite reflecting the laser beam is denoted as S2 (satellite 2).

Inertial Reference Frame (IRF): (*Figure 4-1*)
The inertial reference frame for the NGGM mission is assumed to be realized by the J2000 Equatorial Reference Frame, defined as follows:

- Origin, O_{J2000}, located at the centre of the Earth.
- X_{J2000} axis at the intersection of the mean ecliptic plane with the mean equatorial plane at the date of 01/01/2000 and pointing positively towards the vernal equinox.
- Z_{J2000} axis orthogonal to the mean equatorial plane at the date 01/01/2000.
- Y_{J2000} axis completing a right-handed orthogonal reference frame.

Local Orbital Reference Frame (LORF) referred to the satellite S1 or S2 (*Figure 4-1*, *Figure 4-2*)

- Origin, O_O, located at the satellite COM position.
- X_O (roll) axis parallel to the instantaneous direction of the orbital velocity vector ($\underline{V}$), with the same sign of $\underline{V}$.
- Y_O (pitch) axis parallel to instantaneous direction of the orbital angular momentum ($\underline{N}$), with the same sign of $\underline{N}$ ($\underline{V}$ and $\underline{N}$ are orthogonal by definition, since $\underline{N} = \underline{R} \times \underline{V}$, where $\underline{R}$ is the vector from the Earth centre to O_O).
- Z_O (yaw) axis parallel to $\underline{V}\times\underline{N}$, with the same sign of $\underline{V}\times \underline{N}$.

The origin and axes of the LORF of the satellite Si (i = 1, 2) are denoted as (O_{Oi}, X_{Oi}, Y_{Oi}, Z_{Oi})

Satellite-to-Satellite Reference Frame (SSRF) (*Figure 4-2*, *Figure 4-4*)

- Origin, O_{SS} in the mid point of the line joining the S1, S2 satellite COMs;
- X_{SS} axis along the direction of the line from the COM of S1 to the COM of S2;
- Y_{SS} axis perpendicular to the plane containing by line joining the S1, S2 satellites COMs and the line joining the Earth center to O_L, positive in the same direction of the orbital angular momentum of the S1 orbit;
- Z_{SS} axis completing a right-handed orthogonal reference frame.

Laser Interferometer Reference Frame (LIRF) of the satellite S1 or S2 (*Figure 4-3*, *Figure 4-4*)

- Origin, O_{LI} located at the center of the central polarizing beam splitter (PBS) cube of the laser interferometer;
- X_{LI} axis parallel to the axis of the measurement laser beam outgoing (towards the outer space) from the central polarizing beam splitter cube of the laser interferometer, positive towards the emission direction of the laser beam (this definition is valid for both S1 and S2, since also S2 has the capability of emitting a laser beam);

- Y_{LI} axis orthogonal to X_{LI} in the plane formed by the measurement laser beams entering into and outgoing from the central polarizing beam splitter cube. Y_{LI} is positive in the opposite direction of the measurement laser beam entering in the central polarizing beam splitter cube before being reflected towards the outer space.
- Z_{LI} axis completing a right-handed orthogonal reference frame.

The LIRF is commonly used with reference to the satellite S1. In this case no index is added to the labels of its origin and axes. Otherwise a suffix 2 will be added.

Satellite Reference Frame (SRF) of the satellite S1 or S2 (*Figure 4-2*, *Figure 4-3*, *Figure 4-4*, *Figure 4-5*)

- Origin, O_S located at the satellite COM position;
- X_S, Y_S, Z_S axes parallel to the X_{LI}, Y_{LI}, Z_{LI} axes.

The origin and axes of the SRF of the satellite Si (i = 1, 2) are denoted as (O_{Si}, X_{Si}, Y_{Si}, Z_{Si})

Laser Beam Reference Frame (LBRF) of the satellite S1 (*Figure 4-4*)

- Origin, O_{LB} coincident with the laser beam exit point from the satellite S1 (defined as the intersection of the axis of laser beam outgoing from the central polarizing beamsplitter cube of the laser interferometer with the inner optical surface of the BSM);
- X_{LB} axis parallel to the laser beam axis, positive in the outgoing direction;
- Y_{LB} axis lying in a plane orthogonal to X_{LB}, at the minimum angular separation from Y_{S1}.
- Z_{LB} axis completing a right-handed, orthogonal reference frame.

When the laser beam is not deflected by the Beam Steering Mechanism, the LBRF is aligned to the SRF (ACF) of the satellite S1.

Attitude Control Frame (ACF) of the satellite S1 or S2 (*Figure 4-4*)

- Origin, O_B located at the satellite COM position;
- Unit vectors X_B, Y_B, Z_B mutually perpendicular and directed along the principal inertia axes of the satellite.

The ACF axes are nominally aligned to the SRF axes. The origin and axes of the ACF of the satellite Si (i = 1, 2) are denoted as (O_{Bi}, X_{Bi}, Y_{Bi}, Z_{Bi}).

Accelerometer Reference Frame (ARF) of each accelerometer A_i (*Figure 4-5*, *Figure 4-6*, *Figure 4-7*)

- O_{Ai} located at the centre of the accelerometer A_i (i.e. at the centre of the proof mass)
- X_{Ai}, Y_{Ai}, Z_{Ai} axes parallel to the sensitive axes of the accelerometer A_i

The axes of the ARF are nominally parallel to the axes of the SRF of the satellite on which the accelerometer is installed.

Each proof mass has a less sensible measurement axis (X_{TM}) and two ultrasensible measurement axes (Y_{TM} and Z_{TM}) *Figure 4-6*. The test masses are mounted so that along the X_B axis there are only ultrasensible measurement axes.
The overall accelerometer setup consists of 6 proof masses arranged in pairs, which are positioned along the axes (X_B, Y_B, Z_B) of the Attitude Control Frame (ACF), as shown in *Figure 4-7*. The colored reference frames indicate the orientation of each ARF wrt the ACF axes X_B, Y_B, Z_B.
The distance between the center of each test mass and the satellite COM is 0.1835 m.
For the satellite attitude/drag free control purposes the accelerometer pair 2-5 is exploited.

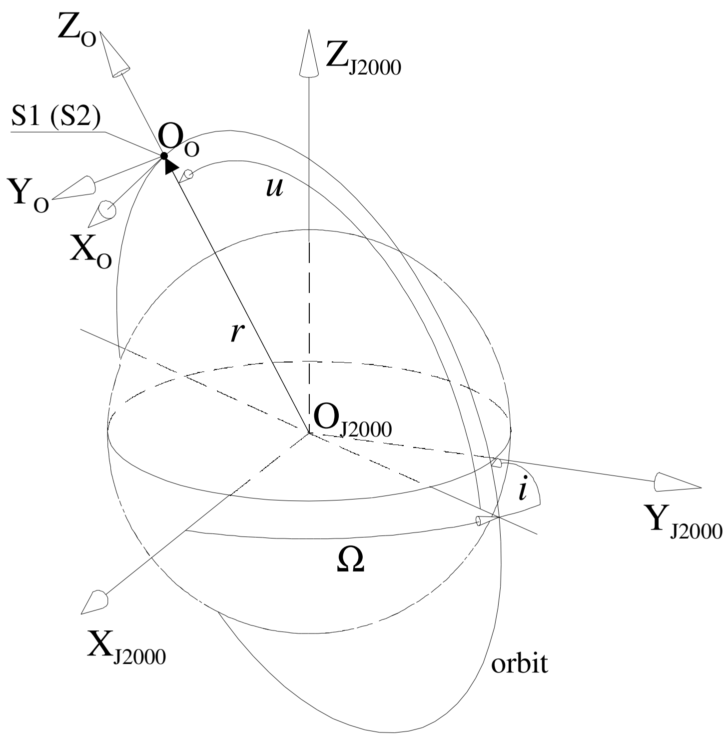

Figure 4-1: IRF and LORF of the satellite S1 (S2): i = orbit inclination, Ω = longitude of the ascending node, u = argument of latitude of the satellite S1 (S2), r = radius vector of the satellite S1 (S2)

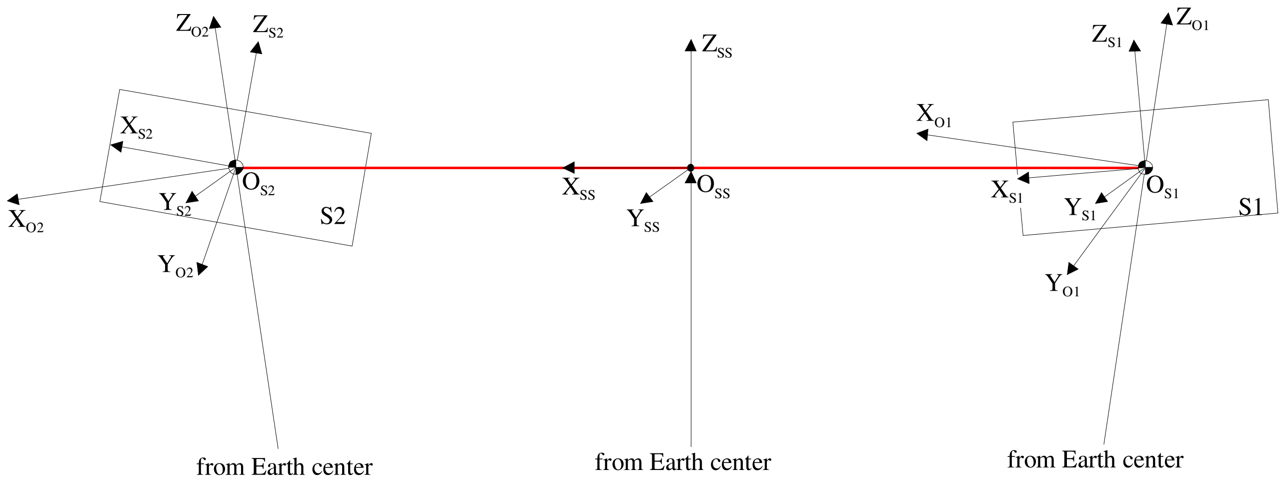

Figure 4-2: Local Orbital Reference Frame, Satellite-to-Satellite Reference Frame and Satellite Reference Frame

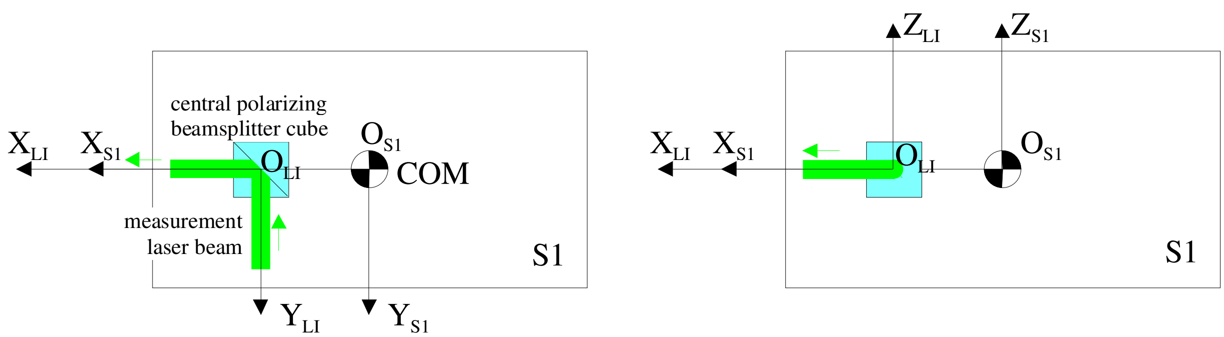

Figure 4-3: Laser Interferometer Reference Frame and Satellite Reference Frame of S1

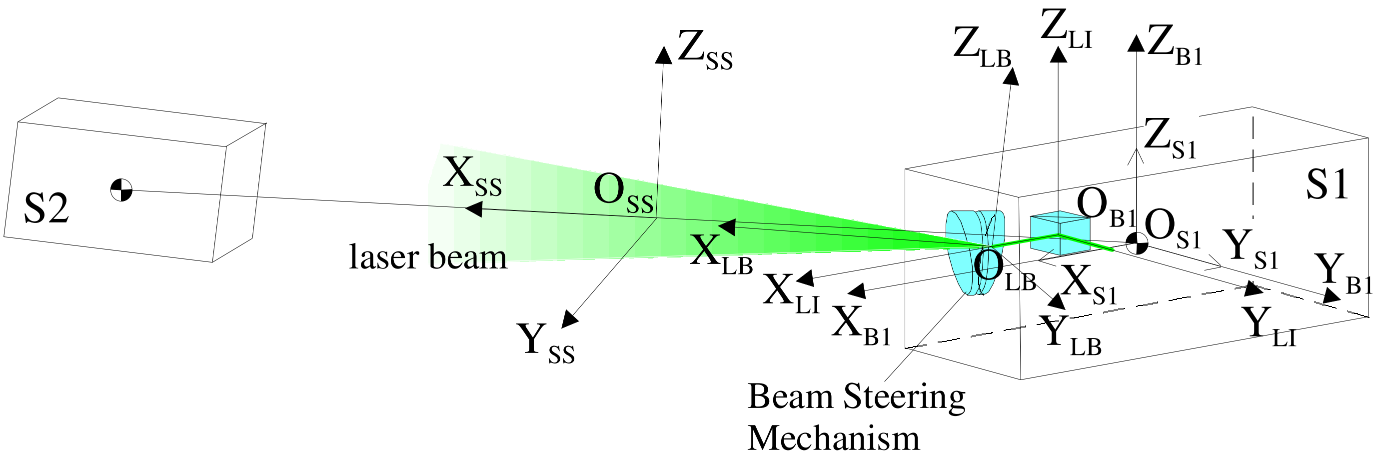

Figure 4-4: Satellite-to-Satellite Reference Frame, Laser Interferometer Reference Frame, Satellite Reference Frame, Laser Beam Reference Frame and Attitude Control Frame

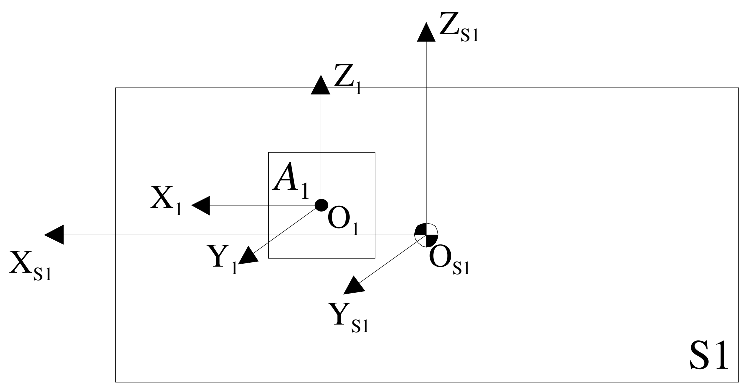

Figure 4-5: Satellite Reference Frame and Accelerometer Reference Frame for the satellite S1

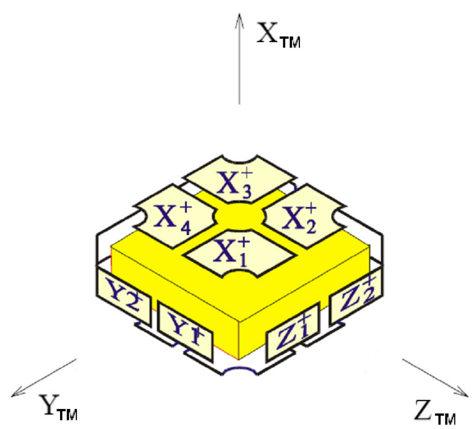

Figure 4-6: Proof mass reference frame

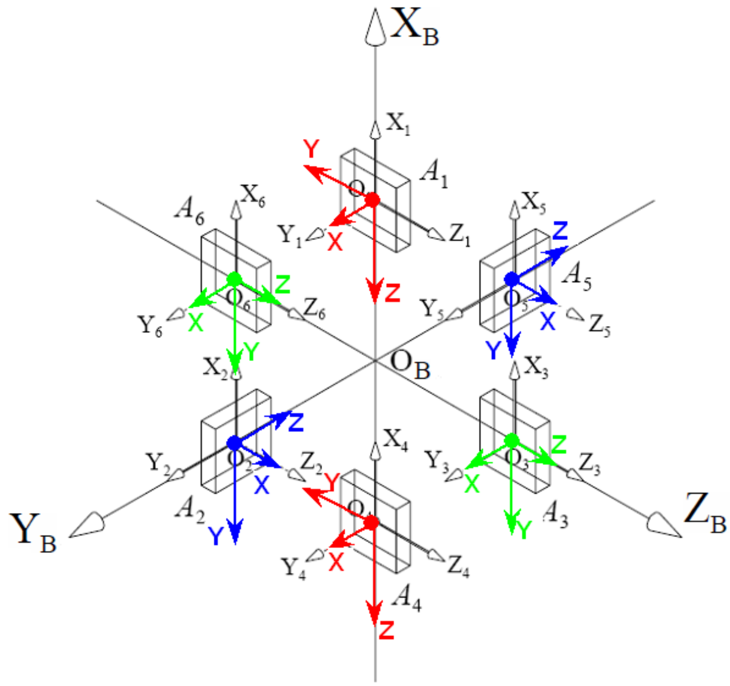

Figure 4-7: Arrangement of the proof masses along the ACF axes. O_B is coincident with the satellite COM. Colored reference frames indicate the orientation of each ARF wrt the ACF axes X_B, Y_B, Z_B.

5. QUATERNIONS

5.1 Quaternion definition

Given two orthogonal, right-handed reference frames, in general the second frame can be obtained from the first one by a single rotation of minimum amplitude θ about a unit vector **e**. Such a rotation is unique and can be represented by the quaternion:

$$\mathbf{q} = q_1\mathbf{i} + q_2\mathbf{j} + q_3\mathbf{k} + q_4$$

where the components (q_1, q_2, q_3, q_4) are defined as follows:

$$q_1 = e_x \sin(\theta/2), \quad q_2 = e_y \sin(\theta/2), \quad q_3 = e_z \sin(\theta/2), \quad q_4 = \cos(\theta/2)$$

where (e_x, e_y, e_z) are the components of the vector **e** (either in the first or second frame, since it is invariant being the rotation axis).

In the following, the first frame will be identified as the Inertial Reference Frame (IRF) and the second one as the Attitude Control Frame (ACF).
The attitude quaternion computed by the E2E simulator dynamics is then exactly the quaternion defined above.
It can be identified as $\mathbf{q}_{IRF,\ ACF}$, and this expression has to be read as *q represents the attitude of ACF w.r.t IRF*.
The rotation is intended to be positive in a counterclockwise direction around the unit vector **e** from IRF to ACF.

5.2 Quaternion corresponding to an elementary rotation

The quaternion corresponding to a rotation of amplitude φ about the x axis is:

$$q_1 = \sin(\varphi/2); \; q_2 = 0; \; q_3 = 0; \; q_4 = \cos(\varphi/2)$$

The quaternion corresponding to a rotation of amplitude θ about the y axis is:

$$q_1 = 0; \; q_2 = \sin(\theta/2); \; q_3 = 0; \; q_4 = \cos(\theta/2)$$

The quaternion corresponding to a rotation of amplitude ψ about the z axis is:

$$q_1 = 0; \; q_2 = 0; \; q_3 = \sin(\psi/2); \; q_4 = \cos(\psi/2)$$

5.3 From quaternion to rotation matrix

The rotation matrix from IRF to ACF, that is the matrix that transforms the components of a vector **v** in the IRF to the components of the same vector in the ACF, can be derived from the quaternion **q** as:

$$
{}_{ACF}[\mathrm{R}]_{\mathrm{IRF}} = \begin{pmatrix} q_1^2 - q_2^2 - q_3^2 + q_4^2 & 2\left(q_1 q_2 + q_3 q_4\right) & 2\left(q_1 q_3 - q_2 q_4\right) \\ 2\left(q_1 q_2 - q_3 q_4\right) & -q_1^2 + q_2^2 - q_3^2 + q_4^2 & 2\left(q_2 q_3 + q_1 q_4\right) \\ 2\left(q_1 q_3 + q_2 q_4\right) & 2\left(q_2 q_3 - q_1 q_4\right) & -q_1^2 - q_2^2 + q_3^2 + q_4^2 \end{pmatrix}
$$

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